Expansion of A Theorem about Triangles

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1. Introduction

In math, theorems can be expanded.

Researches were done about the expanded Pythagorean theorem. → How does the cosine theorem apply in tetrahedrons?

2. Purpose

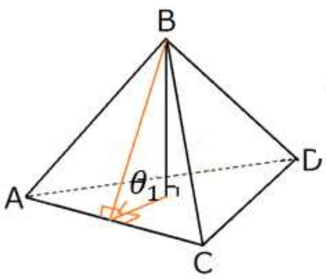
- To research about the cosine applied to tetrahedron
- To prove cosine theorem that compose on tetrahedron

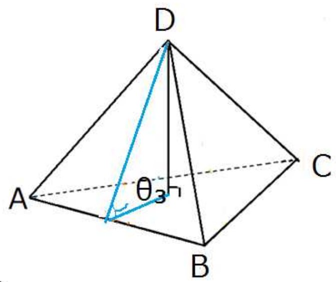
3. Theory

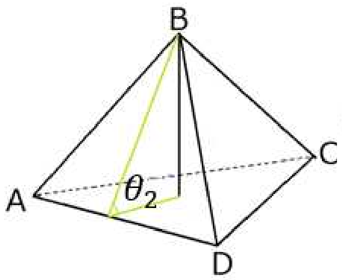
cosine theorem in a triangle
→ relationship between two edges, an angle and the opposite edge

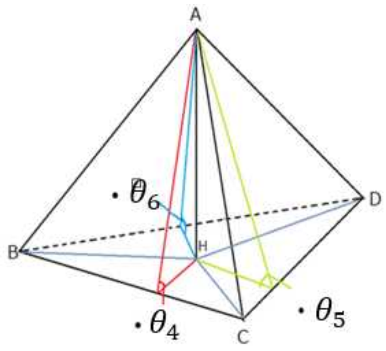
cosine theorem in tetrahedron
→ relationship between three areas, its angles, and the opposite area

4. Prove









$$\Delta\mathrm{BCD} = \Delta\mathrm{HBC} + \Delta\mathrm{HCD} + \Delta\mathrm{HBD}$$

$$\Delta\mathrm{BCD} = \Delta\mathrm{ABC}\cos\theta_4 + \Delta\mathrm{ACD}cos\theta_5 + \Delta\mathrm{ABD}cos\theta_6$$

$$\Delta\mathrm{ABC} = \Delta\mathrm{AB}D\cos\theta_3 + \Delta\mathrm{ACD}cos\theta_1 + \Delta\mathrm{BCD}cos\theta_4$$

$$\Delta\mathrm{ACD} = \Delta\mathrm{AB}C\cos\theta_1 + \Delta\mathrm{ABD}cos\theta_2 + \Delta\mathrm{BCD}cos\theta_5$$

$$\Delta\mathrm{ABD} = \Delta\mathrm{AB}C\cos\theta_3 + \Delta\mathrm{ABD}cos\theta_2 + \Delta\mathrm{BCD}cos\theta_6$$

$$\Delta\mathrm{ABC}=S_1, \Delta\mathrm{ABD}=S_2, \Delta\mathrm{ACD}=S_3, \Delta\mathrm{BCD}=S_4$$

$$S_4 = S_1cos\theta_4 + S_2cos\theta_5 + S_3cos\theta_6$$

$$S_1 = S_2cos\theta_3 + S_3cos\theta_1 + S_4cos\theta_4$$
$$\Leftrightarrow cos\theta_4 = \frac{1}{S_4}(S_1 - S_2cos\theta_3 - S_3cos\theta_1)$$

$$S_2 = S_1cos\theta_1 + S_3cos\theta_2 + S_4cos\theta_5$$
$$\Leftrightarrow cos\theta_5 = \frac{1}{S_5}(S_2 - S_1cos\theta_1 - S_3cos\theta_2)$$

$$S_3 = S_1cos\theta_3 + S_3cos\theta_2 + S_4cos\theta_6$$
$$\Leftrightarrow cos\theta_6 = \frac{1}{S_6}(S_3 - S_1cos\theta_3 - S_3cos\theta_2)$$

$$S_4^2 = S_1^2 + S_2^2 + S_3^2$$
$$-2S_1\ S_2\ cos\theta_1 - 2S_2\ S_3\ cos\theta_2 - 2S_3\ S_1\ cos\theta_3$$

5. Conclusion

- The expansion of the cosine theorem is expressed by

$$S_4^2 = S_1^2 + S_2^2 + S_3^2$$
$$-2S_1\ S_2\ cos\theta_1 - 2S_2\ S_3\ cos\theta_2 - 2S_3\ S_1\ cos\theta_3.$$

6. Future Research

- To research about the tetrahedron that has an orthocenter in it
- To research how the cosine theorem become in n-dimension